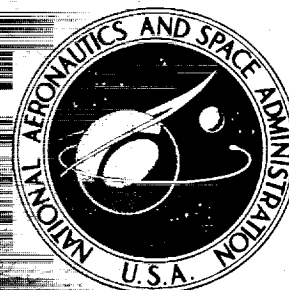


# NASA TECHNICAL REPORT



NASA TR R-458

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## AN EXACT TRANSFORMATION FROM GEOCENTRIC TO GEODETIC COORDINATES FOR NONZERO ALTITUDES

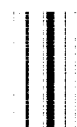
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1976

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# AN EXACT TRANSFORMATION FROM GEOCENTRIC TO GEODETIC COORDINATES FOR NONZERO ALTITUDES

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## INTRODUCTION

In space position measurement systems, it is often necessary to transform geocentric to geodetic coordinates. Consequently, it is important to have an exact closed-form solution for the transformation that is free of embedded singularities. An exact solution is particularly important for determining missile positions. All the previous work in this area falls into two categories: formulas that are mathematically exact but involve equations that become unstable in the neighborhood of their singularities, and formulas that are approximations that may or may not have inherent singularities. References 1 and 2 are of the former type, and references 3 to 5 are of the latter.

Although some of these methods are adequate under some conditions, they have limitations under certain circumstances. This paper presents an exact closed-form solution for the transformation that is free of singularities. The solution permits the evaluation of any of the existing methods under any conditions and should serve as a primary standard.

The appendix contains computer subroutines that implement the ideas that follow and examples of input and output data.

## SYMBOLS

Parenthetical symbols are computer identifiers for variables.

$a$ (A)	major axis of ellipsoid
$b$ (B)	minor axis of ellipsoid
$d$	the distance from any point in space to the ellipsoid
$d_x$	$= x_0 - x_2$

$d_y$	$= y_0 - y_2$
$d_z$	$= z_0 - z_2$
$e$ (E)	eccentricity
$f$ (F)	flattening factor
$f(x, y, z)$	general function to be minimized
$g$	restraint imposed on $f$
$H$	function which is composite of functions $f$ and $g$
$h$ (ALT)	altitude normal to the ellipsoid
$h_s$ (ALTS)	altitude computed from distance formula
$p_0$	any point in space
$p_2$	the point on the ellipsoid that is the minimum distance from the point $p_0$
$t_x$	percentage of error in $d_x$
$t_y$	percentage of error in $d_y$
$t_z$	percentage of error in $d_z$
$X$	coordinate axis that intercepts the Greenwich meridian
$x, y, z$	coordinates of any point on the ellipsoid
$x_0, y_0, z_0$ (X0, Y0, Z0)	coordinates of $p_0$
$x_2, y_2, z_2$ (X2, Y2, Z2)	coordinates of $p_2$
$Y$	coordinate axis that is in a direction normal to the plane determined by the intersection of $X$ and $Z$
$Z$	coordinate axis that intercepts the poles

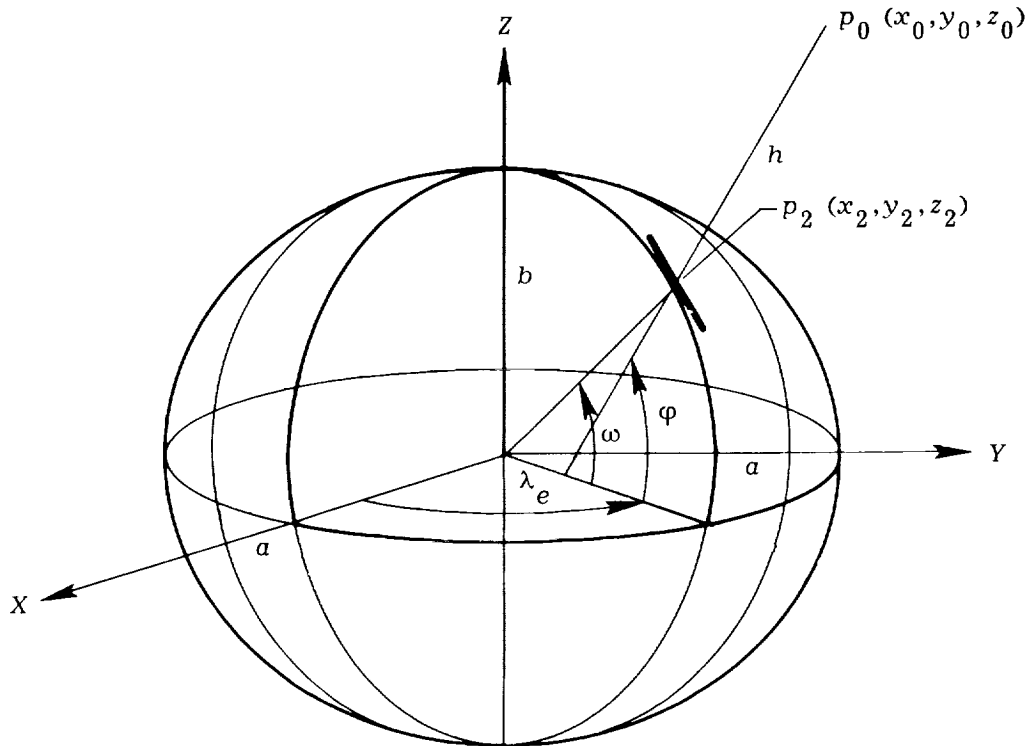
$\alpha$ (V)	arbitrary real constant to be determined
$\varepsilon$	error in $\alpha$
$\lambda_e$ (ALONG)	longitude east
$\lambda_w$ (ALONG)	longitude west
$\varphi$ (ALAT)	geodetic latitude
$\omega$	geocentric latitude

### ANALYTICAL FORMULATION

Let a model of the earth be an ellipsoid given by the following equation (ref. 6):

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \quad (1)$$

where  $a > b$ . In addition, let  $p_0 (x_0, y_0, z_0)$  be any point above the ellipsoid, as shown in the sketch below.



The line perpendicular to the surface of the ellipsoid from  $p_0$  is the altitude of  $p_0$  above the ellipsoid and hence is the shortest distance from  $p_0$  to that surface (eq. (1)).

Therefore, by minimizing the distance from  $p_0$  to the surface of the ellipsoid, it is possible to acquire the coordinates  $(x_2, y_2, \text{ and } z_2)$  of the point on the surface that cause the distance defined by

$$d = \left[ (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right]^{1/2} \quad (2)$$

to be minimum.

The Lagrange multiplier method (ref. 7) is used to implement this minimization where

$$f(x, y, z) = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$$

and

$$g(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1$$

Then

$$H(x, y, z, \alpha) = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - \alpha \left( \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 \right)$$

If partial derivatives are taken in turn and each is equated to zero, the following equations result:

$$\frac{\partial H}{\partial x} = 2(x - x_0) - \frac{\alpha 2x}{a^2} = 0 \text{ or } x = x_0 / (1 - \alpha/a^2) \quad (3)$$

$$\frac{\partial H}{\partial y} = 2(y - y_0) - \frac{\alpha 2y}{a^2} = 0 \text{ or } y = y_0 / (1 - \alpha/a^2) \quad (4)$$

$$\frac{\partial H}{\partial z} = 2(z - z_0) - \frac{\alpha 2z}{b^2} = 0 \text{ or } z = z_0 / (1 - \alpha/b^2) \quad (5)$$

$$\frac{\partial H}{\partial \alpha} = -\frac{x^2}{a^2} - \frac{y^2}{a^2} - \frac{z^2}{b^2} + 1 = 0 \quad (6)$$



Equations (3) to (5) can be substituted into equation (6) as follows:

$$-\left[x_0/(1 - \alpha/a^2)\right]^2/a^2 - \left[y_0/(1 - \alpha/a^2)\right]^2/a^2 - \left[z_0/(1 - \alpha/b^2)\right]^2/b^2 + 1 = 0$$

or

$$-x_0^2(b^2 - 2\alpha + \alpha^2/b^2) - y_0^2(b^2 - 2\alpha + \alpha^2/b^2) - z_0^2(a^2 - 2\alpha + \alpha^2/a^2) + (a^2 - 2\alpha - \alpha^2/a^2)(b^2 - 2\alpha + \alpha^2/b^2) = 0$$

Simplifying and collecting,

$$\begin{aligned} & (1/a^2 b^2) \alpha^4 - 2(1/a^2 + 1/b^2) \alpha^3 + \left(4 + a^2/b^2 + x_0^2/b^2 - y_0^2/b^2 - z_0^2/a^2\right) \alpha^2 \\ & + 2(x_0^2 + y_0^2 + z_0^2 - a^2 - b^2) \alpha + a^2 b^2 - x_0^2 b^2 - y_0^2 b^2 - z_0^2 a^2 = 0 \end{aligned} \quad (7)$$

This is a quartic equation in  $\alpha$  and can be solved in closed form (ref. 8). The proper solution can be determined if the solution is restricted to the real zeros of equation (7) and that zero of equation (7) is chosen that causes  $d$  to be smallest. The appropriate solution of this quartic has no singularities. That is,  $\alpha$  does not approach plus or minus infinity, because that would imply from equations (3) to (5) that  $x_2$ ,  $y_2$ , and  $z_2$  equal zero. Equations (3) to (5) do not become unstable, because the statement  $\alpha = a^2$  or  $\alpha = b^2$  implies that  $x_2$ ,  $y_2$ , or  $z_2$  approaches infinity. Since altitude is nonzero,  $\alpha$  does not equal zero.

Let  $\alpha_0$  be that zero that satisfies the quartic equation. Then, from equations (3) to (5),

$$x_2 = x_0/(1 - \alpha_0/a^2) = x_0 a^2/(a^2 - \alpha_0) \quad (8)$$

$$y_2 = y_0/(1 - \alpha_0/a^2) = y_0 a^2/(a^2 - \alpha_0) \quad (9)$$

$$z_2 = z_0/(1 - \alpha_0/b^2) = z_0 b^2/(b^2 - \alpha_0) \quad (10)$$

$$h_s = \left[(x_2 - x_0)^2 + (y_2 - y_0)^2 + (z_2 - z_0)^2\right]^{1/2} \quad (11)$$

$$\varphi = \sin^{-1}(z_0 - z_2)/h_s \quad (12)$$

$$\lambda_e = \tan^{-1}(y_2/x_2) = \tan^{-1}(y_0/x_0) = -\lambda_w \quad (13)$$

Although the foregoing formulas are mathematically exact, for computational reasons it is usually advantageous to normalize all distances, because this minimizes errors due to rounding off. However, errors due to rounding off are still possible. To circumvent this problem for altitudes not near zero, consider the following error analysis with respect to the computation of  $h_s$ ,  $\lambda$ , and  $\varphi$ .

Let  $\varepsilon$  be a small error introduced in the computation of the zero of the quartic. In addition, let  $d_z = z_0 - z_2 = -z_0\alpha_0/(b^2 - \alpha_0)$ . Then the error in  $d_z$  is introduced by  $\alpha_0$  only, because  $a$ ,  $b$ ,  $x_0$ ,  $y_0$ , and  $z_0$  are all exact. It is easy to show that the percentage of error,  $t_z$ , of  $d_z$  is as follows:

$$t_z = -1 + (1 + \varepsilon/\alpha_0) \left[ (b^2 - \alpha_0)/(b^2 - \alpha_0 - \varepsilon) \right]$$

From equation (10),  $b^2 - \alpha_0 > b^2$ . Therefore  $|b^2 - \alpha_0| > |\alpha_0| > |\varepsilon|$ , and since  $\varepsilon$  is small,  $(b^2 - \alpha_0)/(b^2 - \alpha_0 - \varepsilon) \approx 1$ . Therefore  $t_z \approx \varepsilon/\alpha_0$ . Similarly,  $t_x, t_y \approx \varepsilon/\alpha_0$ .

Since equation (12) involves the ratio of two numbers with virtually equal percentages of error, latitude is undisturbed by a small error in the computation of  $\alpha_0$  due to rounding off.

As noted above, this analysis is valid only if the altitude does not approach zero; otherwise  $\alpha_0$  approaches zero and the value  $|\varepsilon/\alpha_0|$  may approach infinity. The instability of  $|\varepsilon/\alpha_0|$  suggests that equation (12) may become unstable under these conditions.

This final problem can be resolved. If  $p_2 (x_2, y_2, z_2)$  is the point on the ellipsoid that intersects the normal drawn from  $p_0 (x_0, y_0, z_0)$ , the following equation (ref. 6) may be used:

$$\varphi = \tan^{-1} \left[ \tan \omega / (1 - e^2) \right] \quad (14)$$

where  $\tan \omega = z_2 / (x_2^2 + y_2^2)^{1/2}$ . From these relationships,  $\varphi$  may be determined immediately. The error, if any, in  $\tan \omega$  introduced by the computational round-off error in  $\alpha_0$  is inconsequential, because it can easily be shown that the percentage of error in  $x_2$ ,  $y_2$ , and  $z_2$  is virtually zero. In fact, the percentage of error in  $z_2$  and  $x_2$  or  $y_2$  is given by  $(b^2 - \alpha_0)/(b^2 - \alpha_0 - \varepsilon) - 1$  and  $(a^2 - \alpha_0)/(a^2 - \alpha_0 - \varepsilon) - 1$ , respectively. Since  $\varepsilon$  is small for any value of  $\alpha_0$ , the quantities

$(b^2 - \alpha_0)/(b^2 - \alpha_0 - \epsilon)$  and  $(a^2 - \alpha_0)/(a^2 - \alpha_0 - \epsilon)$  approximate 1 under any conditions. Thus, equation (14) is preferable to equation (12) for determining geodetic latitude for any altitude.

However, even the smallest error in  $\alpha_0$  causes more than negligible error in the computation of  $h_s$  (eq. (11)), because the error is amplified by the scaling factor used to restore the true value of  $h$  after computations are completed. This analysis suggests that it is preferable to use the following exact formulas from reference 6 to compute  $h$  instead of equation (11):

$$\begin{aligned}x_0 &= \left[ a / \left( 1 - e^2 \sin^2 \varphi \right)^{1/2} + h \right] \cos \varphi \cos \lambda \\y_0 &= \left[ a / \left( 1 - e^2 \sin^2 \varphi \right)^{1/2} + h \right] \cos \varphi \sin \lambda \\z_0 &= \left[ a (1 - e^2) / \left( 1 - e^2 \sin^2 \varphi \right)^{1/2} + h \right] \sin \varphi\end{aligned}$$

where  $f = (a - b)/a$  and  $e^2 = 2f - f^2$ . Since latitude and longitude are known,  $h$  can be computed directly and the proper equation can be chosen to compute  $h$  to avoid division by values close to zero.

In this way the geodetic coordinates  $\varphi$ ,  $\lambda_w$ , and  $h$  are determined exactly both mathematically and computationally. Therefore, the computational accuracy of the method is the accuracy of the computer used.

#### CONCLUDING REMARKS

A mathematically exact method for computing geodetic coordinates from geocentric coordinates is derived. The computational accuracy achieved by using the method is as accurate as the computer used. The transformation provides a primary standard and makes possible the evaluation of any of the existing methods.

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National Aeronautics and Space Administration  
Edwards, Calif., November 4, 1975*

## APPENDIX

### COMPUTER SUBROUTINES AND DATA SAMPLES

The following computer subroutines implement the theory presented in the text. The subroutines were written in FORTRAN IV.

The comments in the listings should be sufficient for their comprehension and modification. A sample input and output listing is provided to facilitate the verification of the correct FORTRAN code.

## APPENDIX - Continued

### Subroutine GEOD

		SUBROUTINE GEOO(ELE,ELEM,ELES,AZI,AZIM,AZIS,PHI,PHIM,PHIS, *ALM,ALMM,ALMS,RAN,H,ALAT,ALONG,ALT)	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57
	C		
	C		
5	C	THIS SUBROUTINE COMPUTES THE ALTITUDE, GEODETTIC LATITUDE AND LONGITUDE GIVEN THE RANGE, AZIMUTH AND ELEVATION OF A POINT WITH RESPECT TO A RADAR SITE.	
	C		
	C		
	C		
10	C	ELE,ELEM,ELES ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY OF ELEVATION OF THE TARGET WITH RESPECT TO A RADAR SITE.	
	C		
	C	AZI,AZIM,AZIS ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY OF AZIMUTH OF THE TARGET WITH RESPECT TO A RADAR SITE.	
	C		
15	C	PHI,PHIM,PHIS ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY OF THE GEODETTIC LATITUDE OF THE RADAR SITE.	
	C		
	C	ALM,ALMM,ALMS ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY OF THE LONGITUDE OF THE RADAR SITE.	
20	C		
	C	RAN IS THE RANGE OF THE TARGET WITH RESPECT TO THE RADAR SITE. H IS THE ALTITUDE OF THE RADAR SITE ABOVE THE REFERENCE ELLIPSOID.	
	C		
25	C	ALAT IS THE COMPUTED GEODETTIC LATITUDE. ALONG IS THE COMPUTED LONGITUDE. ALT IS THE COMPUTED ALTITUDE.	
	C		
	C	DIMENSION R(12),ROOT(12)	
30	C		
	C	DEFINE MAJOR AND MINOR AXIS OF ELLIPSOID IN FEET.	
	C		
		A=20925832. B=20854892. F=(A-B)/A GE=2*F-F**2	
35			
	C		
	C	NORMALIZE IN UNITS OF A.	
	C		
40		ZP=A A1=A H=H/ZP RAN=RAN/ZP A=A/ZP B=B/ZP GK=3.1415926536 CCN=GK/180.	
	C		
	C	CONVERT ANGLES FROM DEGREES TO RADIANS.	
50			
	C	ELE=ELE+(ELEM/60)+(ELES/3600) PHI=PHI+(PHIM/60)+(PHIS/3600) AZI=AZI+(AZIM/60)+(AZIS/3600) ALM=ALM+(ALMM/60)+(ALMS/3600) ALM=360-ALM PHI=PHI*CON ALM=ALM*CON	
55			

# APPENDIX - Continued

		AZI=AZI*CON	58
		ELE=ELE*CON	59
60	C		60
	C		61
	C	CCMPUTE THE TRANSFORMATION OF TARGET COORDINATES,	62
	C	FROM POLAR COORDINATES TO CARTESIAN COORDINATES,	63
	C	CENTERED AT TRACKING RADAR.	64
65	C		65
		XO=RAN*SIN(AZI)*COS(ELE)	66
		ZO=RAN*SIN(ELE)	67
		YO=RAN*COS(AZI)*COS(ELE)	68
70	C		69
	C	CCMPUTE THE TRANSFORMATION OF THE RADAR SITE GIVEN	70
	C	IN GEODETIC POLAR COORDINATES TO CARTESIAN COORDINATES	71
	C	WITH RESPECT TO THE GEOCENTRIC CENTER OF THE EARTH.	72
	C		73
75		EX=1/(1-CE*(SIN(PHI)**2))	74
		EX=ABS(EX)	75
		EX=EX**.5	76
		EX=EX*A	77
		E0=(EX+H)*COS(FHI)*COS(ALM)	78
		F0=(EX+H)*COS(PHI)*SIN(ALM)	79
80		G0=(EX*(1-CE)+H)*SIN(PHI)	80
	C		81
	C	CCMPUTES THE DIRECTION COSINES OF THE AXIS OF THE RADAR SITE.	82
	C		83
85		U1=-SIN(ALM)	84
		U2=COS(ALM)	85
		U3=0	86
		V1=-COS(ALM)*SIN(PHI)	87
		V2=-SIN(ALM)*SIN(PHI)	88
		V3=COS(PHI)	89
90		W1=COS(ALM)*COS(FHI)	90
		W2=SIN(ALM)*COS(FHI)	91
		W3=SIN(PHI)	92
	C		93
95	C	CCMPUTE THE TRANSFORMATION OF THE POINT OR TARGET	94
	C	FROM CARTESIAN COORDINATES CENTERED AT TRACKING	95
	C	RADAR TO GEOCENTRIC CARTESIAN COORDINATES.	96
	C		97
		X5=XO*U1+YO*V1+ZO*W1+E0	98
		Y5=XO*U2+YO*V2+ZO*W2+F0	99
100		Z5=XO*U3+YO*V3+ZO*W3+G0	100
		X0=X5	101
		Y0=Y5	102
		Z0=Z5	103
105	C		104
	C	CONSTRUCT THE COEFFICIENTS OF THE QUARTIC EQUATION:	105
	C	R(5) * X**4 + R(4) * X**3 + R(3) * X**2 +	106
	C	R(2) * X + R(1) = 0	107
	C		108
110		R(5)=1/((A*B)**2)	109
		R(4)=-2*((1/(A**2))+(1/(B**2)))	110
		R(3)=4+((B/A)**2)+((A/B)**2)-((X0/B)**2)-((Y0/B)**2)-((Z0/A)**2)	111
		R(2)=2*(X0**2+Y0**2+Z0**2)-2*(A**2+B**2)	112
		R(1)=(A*B)**2-(X0*B)**2-(Y0*B)**2-(Z0*A)**2	113
	C		114

# APPENDIX - Continued

115	C	CALL QUART TO SOLVE THE QUARTIC EQUATION FOR ALL	115
	C	REAL ZEROS WHERE:	116
	C	R = ARRAY OF COEFFICIENTS	117
	C	ROOT = ARRAY OF REAL ZERCS	118
	C	NI = NUMBER OF REAL ZERCS	119
120	C		120
	C	CALL QUART(R,ROOT,NI)	121
	C		122
	C	DETERMINE THE COORDINATES ON THE ELLIPSOID THAT	123
	C	MAKES THE DISTANCE FROM THE TARGET TO THE SURFACE A MINIMUM.	124
125	C		125
		PS=10.**30	126
		XJ = 0.0	127
		YJ = 0.0	128
		ZJ = 0.0	129
130		DO 16 J=1,NI	130
		V=ROOT(J)	131
		X2=(X0*A**2)/(A**2-V)	132
		Y2=(Y0*A**2)/(A**2-V)	133
		Z2=(Z0*B**2)/(B**2-V)	134
135		XA=(-X0*V)/(A**2-V)	135
		YA=(-Y0*V)/(A**2-V)	136
		ZA=(-Z0*V)/(B**2-V)	137
		U=XA**2+YA**2+ZA**2	138
		U=U**.5	139
140		IF(U.GT.PS) GO TO 16	140
		PS=U	141
		XJ=X2	142
		YJ=Y2	143
		ZJ=Z2	144
145	16	CONTINUE	145
		X2=XJ	146
		Y2=YJ	147
		Z2=ZJ	148
	C		149
150	C	THE ALTITUDE EQUALS THE MINIMUM DISTANCE FROM THE	150
	C	TARGET TO THE SURFACE.	151
		CCN=180./GK	152
	C		153
	C	DETERMINE THE LONGITUDE.	154
155	C		155
		ALONG=ATAN2(Y0,X0)	156
	C		157
	C	DETERMINE THE GEODETIC LATITUDE.	158
	C		159
160		D=X2**2+Y2**2	160
		D=D**.5	161
		AA=D*(1.-CE)	162
		ALAT=ATAN2(Z2,AA)	163
		ALAT=SIGN(ALAT,Z0)	164
165		X1=X0*ZP	165
		Y1=Y0*ZP	166
		Z1=Z0*ZP	167
		EX=1./(1.-CE*(SIN(ALAT)**2))	168
		EX=ABS(EX)	169
170		EX=EX**.5	170
		EX=EX*A1	171

# APPENDIX - Continued

	C		172
	C	COMPUTE ALTITUDE BY AN EXACT EQUATION WHICH MINIMIZES ROUND OFF	173
	C	ERRORS.	174
175	C		175
		IF (ABS(SIN(ALAT)).GT..1)GO TO 14	176
		IF (ABS(COS(ALONG)).GT..1)GO TO 15	177
		ALT=(Y1/(COS(ALAT)*SIN(ALONG)))-EX	178
		GC TO 710	179
180	14	ALT=(Z1/SIN(ALAT))-(EX*(1-CE))	180
		GC TO 710	181
	15	ALT=(X1/(COS(ALAT)*COS(ALONG)))-EX	182
	710	CONTINUE	183
		ALAT=ALAT*CON	184
185	C		185
	C	CHANGE SIGN OF LONGITUDE TO CONFORM WITH INPUT WHICH	186
	C	WAS IN DEGREES WEST...CALCULATIONS GIVE LONGITUDE IN	187
	C	DEGREES EAST, CHANGE SIGN TO GET LONGITUDE IN DEGREES	188
	C	WEST, CONVERT FROM RADIANS TO DEGREES.	189
190	C		190
		ALONG=-ALONG*CCN	191
	C		192
		RETURN	193
		END	194



# APPENDIX - Continued

## Subroutine QUART

```

SUBROUTINE QUART(R,ROOT,NI)
C
C THIS ROUTINE SOLVES FOR THE REAL ZEROS ONLY VIA
C FERRARI'S METHOD. (NEW FIRST COURSE IN THE THEORY OF
5 C EQUATIONS, DICKSON, PP. 51-52)
C
C
C
C R IS THE ARRAY WHICH CONTAINS THE COEFFICIENTS OF THE
10 C QUARTIC ARRANGED IN ASCENDING ORDER.
C
C  $R(5) * X^4 + R(4) * X^3 +$ 
C  $R(3) * X^2 + R(2) * X + R(1) = 0$ 
C
C THE ARRAY ROOT WILL CONTAIN THE REAL ZEROS OF THE QUARTIC
15 C
C NI IS THE SCALAR VARIABLE THAT STATES THE NUMBER OF REAL ZEROS
C
C
C
20 C DIMENSION R(1),ROOT(1)
C NI=0
C
C NORMALIZE COEFFICIENTS SO:
C  $X^4 + B*X^3 + C*X^2 + D*X + E = 0$ 
25 C
C DO 13 J=1,5
19 R(J)=R(J)/R(5)
C B=R(4)
C C=R(3)
30 C D=R(2)
C E=R(1)
C
C CALCULATE COEFFICIENTS B1, C1, D1, OF THE RESOLVENT CUBIC
35 C  $Y^3 + B1*Y^2 + C1*Y + D1 = 0$ 
C
C B1=-C
C C1=B*D-4*E
C D1=-(B**2)*E+4*C*E-D**2
C
C IN SOLVING CUBIC EQUATION, CALCULATE COEFFICIENTS
40 C OF THE CORRESPONDING REDUCED CUBIC WHICH HAS NO TERM
C OF THE SECOND DEGREE BY SETTING  $Y = Z - B1/3$ 
C  $Z^3 + P*Z + Q = 0$ 
C
C P=C1-((B1**2)/3)
45 C Q=D1-((B1*C1)/3)+((2*(B1**3))/27)
C
C THE DISCRIMINANT OF THE GENERAL CUBIC EQUATION IS
C EQUAL TO THE DISCRIMINANT DEL OF THE CORRESPONDING
50 C REDUCED EQUATION.
C
C DELT=18*B1*C1*D1-4*(B1**3)*D1+(B1*C1)**2
C DELV=-4*(C1**3)-27*(D1**2)
C DEL=DELT+DELV
55 C
C IF DEL IS NEGATIVE, ONE ROOT IS REAL, AND TWO ARE
C CONJUGATE IMAGINARIES.
C

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# APPENDIX - Continued

	IF (DEL.GE.0.) GO TO 12	252
	RS=(P/3)**3+(Q/2)**2	253
60	RS=ABS(RS)	254
	A=(((-Q/2)+RS**.5)	255
	W=(((-Q/2)-(RS**.5))	256
	S=ABS(A)	257
	T=ABS(W)	258
65	W1=W	259
	A1=A	260
	VV=1./3.	261
	A=S**VV	262
	W=T**VV	263
70	A=A*SIGN(1.,A1)	264
	W=W*SIGN(1.,W1)	265
	C	266
	C Y1 IS THE SINGLE REAL ROOT OF THE REDUCED CUBIC,	267
	C Y IS THE SINGLE REAL ROOT OF THE GENERAL CUBIC.	268
75	C	269
	Y1=A+W	270
	Y=Y1-(B1/3)	271
	GO TO 10	272
	12 CONTINUE	273
80	C	274
	C IF DEL IS POSITIVE, THERE ARE THREE DISTINCT REAL	275
	C ROOTS. IF DEL IS ZERO, THERE ARE AT LEAST TWO EQUAL	276
	C REAL ROOTS.	277
	C TRIGONOMETRIC SOLUTION IS USED.	278
85	C	279
	TN=(-4*P/3)	280
	TN=ABS(TN)	281
	TN=TN**.5	282
	COS3A=-.5*Q*((-3/P)**1.5)	283
90	IF (ABS(COS3A).GT.1.) COS3A=SIGN(1.,COS3A)	284
	ARC=ACOS(COS3A)/3	285
	COSA=COS(ARC)	286
	Y1=TN*COSA	287
	Y=Y1-(B1/3)	288
95	10 CONTINUE	289
	C	290
	C BACK TO SOLVING THE QUARTIC, WHERE Y IS SUCH THAT	291
	C $A2X^2 + B2X + C2$ IS THE SQUARE OF A LINEAR FUNCTION,	292
	C $MX + N$ , AND ALSO EQUAL TO $(X^2 + BX/2 + Y/2)^2$	293
100	C	294
	A2=.25*(B**2)-C+Y	295
	B2=.5*B*Y-D	296
	C2=.25*(Y**2)-E	297
	C	298
105	IF (A2.NE.0.) GO TO 111	299
	C	300
	C IN CASE A2=0, $(X^2 + BX/2 + Y/2)^2 = B2X + C2$	301
	C SINCE THE POLYNOMIAL IS A PERFECT SQUARE, B2 = 0	302
	C AND THE QUADRATICS TO BE SOLVED ARE:	303
110	C $X^2 + BX/2 + Y/2 - C2**.5 = 0$ AND	304
	C $X^2 + BX/2 + Y/2 + C2**.5$	305
	C	306
	DEL1 = (B/2)**2 - 4*(Y/2 - C2**.5)	307
	DEL2 = (B/2)**2 - 4*(Y/2 + C2**.5)	308

# APPENDIX - Continued

115	DX1=B/2	309
	DX2=B/2	310
	GO TO 223	311
	111 CONTINUE	312
	C	313
120	C IN CASE A2 IS NOT 0, THEN $P = A2^{*.5}$ , $N = B2/(2*M)$ .	314
	C THEN THE QUADRATICS TO BE SOLVED ARE:	315
	C $X^{*2} + (B/2 - M)*X + Y/2 - N = 0$ AND	316
	C $X^{*2} + (B/2 + M)*X + Y/2 + N = 0$	317
	C	318
125	A2=ABS(A2)	319
	AM=A2**2	320
	AN=B2/(2*AM)	321
	DX1=.5*B-AM	322
	DX2=.5*B+AM	323
130	DEL1 = (B/2 - AM)**2 - 4*(Y/2 - AN)	324
	DEL2 = (B/2 + AM)**2 - 4*(Y/2 + AN)	325
	223 CONTINUE	326
	C	327
	C ROOTS OF TWO QUADRATICS ARE THE FOUR ROOTS OF THE QUARTIC,	328
135	C ONLY THE REAL ROOTS ARE RETURNED.	329
	C	330
	IF(DEL1.LT.0.)GO TO 22	331
	NI=NI+1	332
	ROOT(NI)=(-DX1/2)+((DEL1**2)**.5)/2)	333
140	NI=NI+1	334
	ROOT(NI)=(-DX1/2)-((DEL1**2)**.5)/2)	335
	22 IF(DEL2.LT.0.)GO TO 25	336
	NI=NI+1	337
	ROOT(NI)=(-DX2/2)+((DEL2**2)**.5)/2)	338
145	NI=NI+1	339
	ROOT(NI)=(-DX2/2)-((DEL2**2)**.5)/2)	340
	25 CONTINUE	341
	RETURN	342
	END	343

# APPENDIX - Concluded Input and Output Data Samples

INPUT-----

POINT COORDINATES WRT RADAR SITE		
ELEVATION DEG.		90.0000
AZI DEG.		-0.0000
RANGE FEET		.1000

RADAR SITE COORDINATES		
GEOCENTRIC LATITUDE		55.0000
LONGITUDE		22.0000
ALTITUDE FEET		-0.0000

CUTPUT----

POINT COORDINATES			
GEOCENTRIC LATITUDE		55.0000	.5500000000006E+02
LONGITUDE		22.0000	.219999999988E+02
ALTITUDE FEET		.0999	.998557806015E-01

INPUT-----

POINT COORDINATES WRT RADAR SITE		
ELEVATION DEG.		90.0000
AZI DEG.		-0.0000
RANGE FEET		50000.0000

RADAR SITE COORDINATES		
GEOCENTRIC LATITUDE		44.0000
LONGITUDE		121.0000
ALTITUDE FEET		500.0000

CUTPUT----

POINT COORDINATES			
GEOCENTRIC LATITUDE		44.0000	.4400000000264E+02
LONGITUDE		121.0000	.120999999999E+03
ALTITUDE FEET		50499.9900	.504999899952E+05

INPUT-----

POINT COORDINATES WRT RADAR SITE		
ELEVATION DEG.		90.0000
AZI DEG.		-0.0000
RANGE FEET		99999999.0000

RADAR SITE COORDINATES		
GEOCENTRIC LATITUDE		33.0000
LONGITUDE		88.0000
ALTITUDE FEET		700.0000

CUTPUT----

POINT COORDINATES			
GEOCENTRIC LATITUDE		33.0000	.329999999979E+02
LONGITUDE		88.0000	.879999999988E+02
ALTITUDE FEET		100000099.0492	.100000069905E+10

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